

# Measurement of forward photon-energy spectra for $\sqrt{s} = 13$ TeV proton-proton collisions with the LHCf detector

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## Abstract

The inclusive energy spectra of forward photons in the pseudorapidity regions of  $\eta > 10.94$  and  $8.99 > \eta > 8.81$  measured by the LHC forward (LHCf) experiment with proton-proton collisions at  $\sqrt{s} = 13$  TeV are reported. The results from the analysis of  $0.191 \text{ nb}^{-1}$  of data obtained in June 2015 are compared with the MC predictions of several hadronic interaction models that are used in air-shower simulations for ultra-high-energy cosmic rays. Although none of the models agree perfectly with the data, EPOS-LHC shows the best agreement with the experimental data among the models.

**Keywords:** Large Hadron Collider, Ultra-high energy cosmic-ray, Hadronic

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## 1. Introduction

Hadronic interaction models play an important role in Ultra-High Energy Cosmic-Ray (UHECR) observations. They are used in Monte-Carlo (MC) simulations of air-shower developments induced by UHECRs, which is one of the key tools for reconstructing information about primary cosmic rays from observables measured by ground-based detectors. Currently the Pierre Auger Observatory [1] and the Telescope Array [2] are running for observations of UHECRs. Although the experiments published the results of measured observables which are sensitive to the chemical composition of UHECRs, they have no clear conclusion yet because of the uncertainty related to the choice of a hadronic interaction model [3, 4, 5]. Since it began operating in 2009, the Large Hadron Collider (LHC), the world's largest hadron collider, has provided unique opportunities for testing hadronic interaction models with collision energies exceeding  $10^{15}$  eV in the laboratory frame. The major models used in air-shower simulations for UHECRs were re-tuned and updated by taking into account several experimental results obtained from proton-proton collisions with the center-of-mass collision energies of 0.9 TeV and 7 TeV. These models, QGSJET II-04 [6], EPOS-LHC [7], and SIBYLL 2.3 [8], are called the post-LHC models. However, even with these post-LHC models, inconsistencies between observed data and MC simulations were reported [9].

The LHC forward (LHCf) experiment [10], one of the LHC experiments designed to test hadronic interaction models, had an operation during the early phase of the LHC operation with proton-proton collisions at  $\sqrt{s} = 13$  TeV in 2015. In this paper, we report the results of photon analyses performed on the taken data. The results of photon analyses for the lower-energy collisions of  $\sqrt{s} = 0.9$  TeV and  $\sqrt{s} = 7$  TeV were published in Ref. [11, 12]. Thanks to nearly a factor of two higher collision energy than 7 TeV, the collision energy in the laboratory frame,  $0.9 \times 10^{17}$  eV, was about a factor of four higher, and

the coverage of the transverse momentum  $p_T$  of the measurement was a factor of two wider than the  $p_T$  coverage of the measurement at  $\sqrt{s} = 7$  TeV.

The LHCf has two sampling and imaging calorimeter detectors which are installed both sides of the LHC interaction point IP1 [13]. Each of the two detectors, Arm1 and Arm2, has two calorimeter towers with the acceptances of  $20 \text{ mm} \times 20 \text{ mm}$  and  $40 \text{ mm} \times 40 \text{ mm}$  (Arm1) and  $25 \text{ mm} \times 25 \text{ mm}$  and  $32 \text{ mm} \times 32 \text{ mm}$  (Arm2). In nominal operations, the smaller towers cover the pseudorapidity range above 10, including the zero degree of collisions. The other towers cover the slightly off-center region where  $8.5 < \eta < 9.5$ . Before the operation in 2015, the detectors had been upgraded to improve their radiation hardness by replacing the plastic scintillators with  $\text{Gd}_2\text{SiO}_5$  (GSO) scintillators [14] and the X-Y scintillating-fiber hodoscopes with X-Y GSO bar-bundle hodoscopes [15]. The silicon detector inserted in the Arm2 detector had been upgraded also to optimize the performance. The performance of the upgraded detectors was studied in two beam tests at CERN-SPS before and after the operation at the LHC. We confirmed that the energy resolution and the position resolution for electromagnetic showers were better than the requirements of  $< 5\%$  and  $< 200 \mu\text{m}$ , respectively [16].

## 2. Data

The experimental data used in this analysis were obtained by an LHCf run from 22:32 to 1:30 (CEST) on June 12-13, 2015 during proton-proton collisions at  $\sqrt{s} = 13$  TeV. The operation period corresponded to the first three hours of the LHC Fill 3855, which was one of the low-luminosity LHC runs operated with smaller numbers of bunches and higher  $\beta^*$  of 19 m than the LHC's nominal condition. In the fill, 29 bunches collided at IP1 with a half crossing angle of  $145 \mu\text{rad}$ . Additionally, six and two non-colliding bunches at IP1 circulated for the clockwise and the counter-clockwise beams, respectively. The total luminosity of the colliding bunches during data acquisition was measured by the ATLAS experiment at  $L = (3 - 5) \times 10^{28} \text{ cm}^{-2}\text{s}^{-1}$  [17]. The number of collisions per

bunch crossing,  $\mu$ , was in the range of 0.007 to 0.012. Considering about 15% acceptance of the detectors for inelastic collisions, the pile up of events on a detector was negligible in this analysis.

The recorded total integral luminosity was  $0.191 \text{ nb}^{-1}$  after correction of the data-acquisition live time. Assuming the inelastic cross-section of  $\sigma_{\text{inela}} = 78.5 \text{ mb}$ , it corresponds to  $1.50 \times 10^7$  inelastic collisions. The numbers of the recorded shower events in Arm1 and Arm2 are 17.9 M and 21.0 M, respectively. The trigger efficiency was 100% for photons at greater than 200 GeV.

### 3. MC Simulation

A full MC simulation was performed to obtain some parameters and correction factors used in this analysis and to validate the analysis method. The simulation consisted of the three parts: 1) event generation of p-p inelastic collisions at IP1; 2) particle transportation from IP1 to the front of the detector; and 3) the detector response. These parts were implemented with MC simulation packages Cosmos 7.633 [18] and EPICS 9.15 [19]. In the first part of the simulation, either QGSJET II-04 or EPOS-LHC was used as an event generator and the DPMJET 3.04 [20] model was used as a hadronic interaction model in the detector simulation of the third part. We generated  $10^8$  events with the QGSJET II-04 model. The data set was used as a template sample for particle identification (PID) correction and a training sample for the unfolding method described in Sec. 4.2. Another full MC simulation data set of  $5 \times 10^7$  events was generated with EPOS-LHC and used to validate the analysis method and to estimate systematic uncertainties.

Additionally, we generated  $10^8$  events of inelastic p-p collisions with each hadronic interaction models, EPOS-LHC, QGSJET II-04, DPMJET 3.06, SIBYLL 2.3, and PYTHIA 8.212 [21], using either the PYTHIA dedicated generator or CRMC 1.6.0, an interface tool of event generators [22]. These event sets were used only in Sec. 6 to compare the photon-energy spectra of data and model predictions.

## 4. Analysis

### 4.1. Event Reconstruction

This analysis used an event reconstruction algorithm resembling the one employed in Ref. [12, 23]. The detector upgrades dictated the renewal of calibration parameters by beam tests [15, 16]. Then, the criteria in this analysis were re-optimized by MC simulation studies. We selected the events that meet the criteria of PID for photons and rejection of multi-hit events in which two or more particles hit a calorimeter tower.

The reconstructed energy of each event was rescaled by a factor obtained from a study of  $\pi^0$  events in which photon pairs were detected by the two calorimeter towers of each detector. The invariant mass of a photon pair was calculated using both the measured photon energies and hit positions, assuming that the decay vertex coincides with IP1. The distribution of the reconstructed mass had a peak corresponding to the  $\pi^0$  mass. We compared the peak masses from the data and the MC simulations and obtained energy rescale factors of +3.5% and +1.6% for Arm1 and Arm2, respectively. The factors were consistent with the systematic uncertainty of energy-scale calibrations discussed in Sec. 5.1.

In this analysis, we defined two analysis regions, A and B. Region A is the area of a half-circle shape with  $R < 6$  mm and  $\Delta\phi < 180^\circ$ , where  $R$  is the distance from the beam center and  $\Delta\phi$  is the azimuthal interval on each detector plane. The beam center was defined as the projection of the beam direction at IP1 on the detector surface. Region B is the sector-shape area for which  $35\text{mm} < R < 42\text{mm}$  and  $\Delta\phi < 20^\circ$ . Regions A and B correspond to the pseudorapidity regions for which  $\eta > 10.94$  and  $8.81 < \eta < 8.99$ , respectively. Only the events for which the reconstructed hit-positions were within these two regions were used in the final results. The position resolution of less than 0.2 mm is good enough to neglect effect of event migrations between the inside and the outside of the regions.

### 4.2. Corrections

- Beam-related background

The main contribution of background events is due to interactions between the circulating beams and residual gas in the beam pipe. The background was estimated by using the events associated with non-crossing bunches at IP1. These events were generated purely from the beam-gas interactions. However, the events associated with the colliding bunches were related to both the signal and the background. The estimated background-to-signal ratio was less than 1%; this ratio was subtracted from the measured spectra.

- PID correction

The corrections related to the PID selection, inefficiency of the photon selection and the contamination of hadrons, were performed by a template-fit method of the distribution of the PID estimator,  $L_{90\%}$ , defined as the longitudinal depth, in units of radiation length ( $X_0$ ), at which the integral of the energy deposition in a calorimeter reached 90% of the total. As a criterion of the selection of the photon component, we set an energy-dependent criterion  $L_{90\%,thr}$ , which was defined the  $L_{90\%}$  value to keep 90% efficiency of the photon selection in MC simulations. Figure 1 shows the  $L_{90\%}$  distribution of Arm1-Region A for the reconstructed energy range between 1.1 TeV and 1.2 TeV. The red and the blue lines in Fig. 1, obtained from the MC simulation data set of QGSJET II-04, indicate the template distributions for pure photon and pure hadron samples, respectively. These distributions were produced with normalization obtained from the template-fit result.

- Multi-hit correction

Mis-reconstruction of multi-hit events as single-hit events makes measured spectra harder. Thus, the algorithm and criteria for multi-hit event rejections were adjusted to maximize the efficiency of multi-hit detection by maintaining a reasonably low incidence of mis-reconstructions of single-hit events as multi-hit events. We achieved multi-hit detection efficiencies exceeding 85% across the whole energy range and almost 100% in the

energy range above 2 TeV. About 4% of the total triggered events were identified as multi-hit events. Inefficiencies of the single-hit event selections and contamination of multi-hit events in the event selection were corrected based on the energy-dependent factor obtained from the MC simulation data set of QGSJET II-04. Another correction was performed to recover the flux of photons rejected as multi-hit events. The correction factor was also derived from the MC simulation. Overall, the correction factor ranged within  $\pm 50\%$ , which was the largest in the corrections and was dependent on the choice of an event-generation model in the MC simulation. Multi-hit corrections were performed in the unfolding algorithm described below.

- Spectrum unfolding

We corrected for detector biases in the obtained spectra by performing an unfolding technique based on the iterative Bayesian method [24] provided by the RooUnfold package [25]. The MC simulation data set with  $10^8$  inelastic collisions generated by the QGSJET II-04 model was used as a training sample.

## 5. Systematic Uncertainties

We considered the following contributions as systematic uncertainties of photon spectra. Figure 2 shows the estimated systematic uncertainties for each detector and each region as a function of photon energy.

### 5.1. Energy scale

Energy scale error are attributable to a) the absolute gain calibration of each sampling layer, b) uniformity, c) relative gain calibration of photomultiplier tubes (PMTs) used for readout of scintillator lights, and d) the Landau-Pomeranchuk-Migdal (LPM) effect [26, 27]. The first two contributions were studied in beam tests and are described in Ref. [16]. The third source of errors

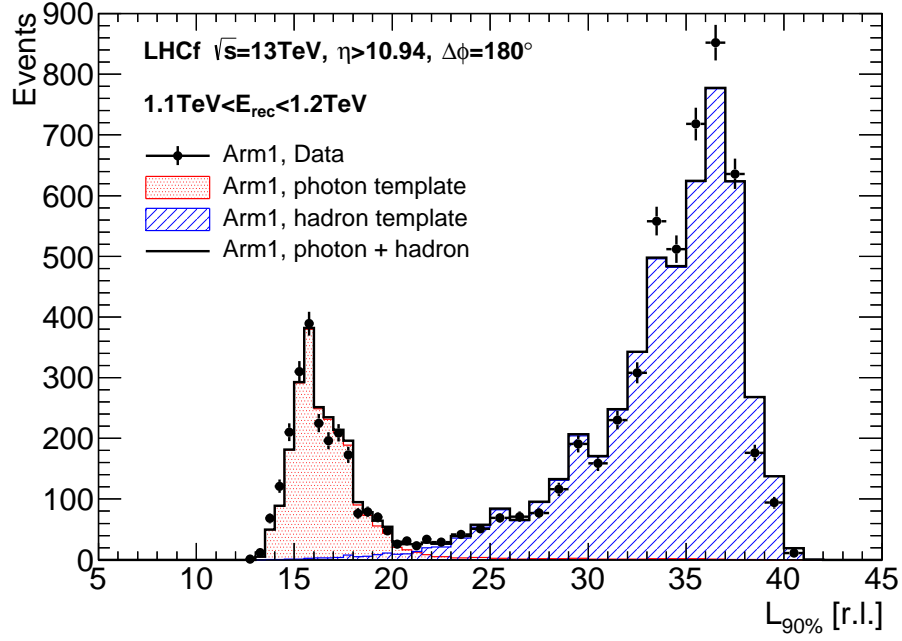


Figure 1: The  $L_{90\%}$  distribution in Arm1 for the events with the reconstructed energy between 1.1 TeV and 1.2 TeV. The black points show the experimental data with the statistical error bars. The red and the blue colored lines correspond to the template distributions obtained from the MC simulation for photons and neutrons, respectively. The black line shows the total of the template distributions. These distributions were normalized by the results of the template fitting.

is related to the differences of high-voltage configurations of PMTs between the beam tests and the operation. The error was about 1.9%. The contribution to the error from the LPM effect was estimated as 0.7% by comparing the detector responses upon activation and inactivation of the LPM effect in the detector simulation. The total energy-scale error, estimated from the quadratic summation of all contributions, was  $\pm 3.4\%$  for Arm1 and  $\pm 2.7\%$  for Arm2. The systematic uncertainty of the spectra was estimated by shifting the energy scale within the errors.



### 5.2. Beam-center stability

The beam center, an important parameter for defining analysis regions, was calculated from the measured hit-map distribution of the hadronic shower events, which were selected such that  $L_{90\%} > L_{90\%,thr}$ . The fluctuations were found to be of the order of 0.3 mm, which was greater than the precision of the mean beam-center measurements that used all data in the fill. The systematic uncertainty associated with the beam-center determination was estimated by artificially moving the beam-center position by  $\pm 0.3$  mm on the x- and y-axes. The measured spectra with the shifted beam-center positions were compared to the original spectrum and the variation was deemed to be the systematic uncertainty.

### 5.3. PID

The systematic uncertainty associated with the PID correction was estimated by changing the criterion for the choice of  $L_{90\%,thr}$  to discriminate between photons and hadrons, as discussed above. Instead of choosing  $L_{90\%,thr}$  to obtain a 90% photon selection efficiency, the PID selection and correction were also performed using the threshold values that produced photon-selection efficiencies of 85% and 95%. We compared the spectra after the correction and determined the systematic uncertainty from the relative deviation from the original spectrum.

### 5.4. Multi-hit identification efficiency

The correction factors attributable to multi-hit event rejections were obtained from the MC simulation. Thus, we tested the consistency of the multi-hit identification efficiencies exhibited by the data and the MC simulation by using “artificial” multi-hit event sets. The artificial multi-hit events were created by merging two independent single-hit events. The combinations of single-hit events were selected to represent the distributions of photon-pair energies and hit-position distances in the true multi-hit events of QGSJET II-04. The same

procedure was performed for the MC simulation to avoid a bias due to the construction method of the sample. We compared the efficiencies exhibited by the data and the MC simulation by using the samples and found inconsistencies of less than approximately 5% and 10% for Arm1 and Arm2, respectively. Finally, the systematic uncertainty of the spectra was calculated by multiplying the relative error of the multi-hit identification efficiency by the ratio of multi-hit events to single-hit events.

### 5.5. Unfolding

The interaction model dependency of the multi-hit correction factors, computed from the training sample, provided to be the main source of systematic uncertainty in the spectrum unfolding. EPOS-LHC predicted a higher multiplicity of photons than QGSJET II-04. Thus, a larger correction factor was expected in EPOS-LHC than in QGSJET II-04. We performed the spectrum unfolding with a training sample of  $5 \times 10^7$  inelastic collisions generated by EPOS-LHC. The relative difference between the QGSJET II-04 and the EPOS-LHC results was chosen as the systematic uncertainty associated with the unfolding.

## 6. Results

Figure 3 presents the photon-energy spectra measured by the Arm1 and the Arm2 detectors. The error bars and the hatched areas indicate the statistical and systematic uncertainties, respectively. In this comparison of the results of the two detectors, the detector-correlated systematic uncertainties due to luminosity and unfolding were not considered. We found general agreement, within the given uncertainties, between the results of the two detectors.

We combined the results using the same method as the analysis presented in Ref. [28]. This approach assumed that the systematic uncertainties of the energy scale, PID correction, performance of multi-hit identification, and beam position showed both bin-by-bin correlation and Arm1-Arm2 noncorrelation. The other systematic uncertainties - luminosity and unfolding - were assumed to be fully

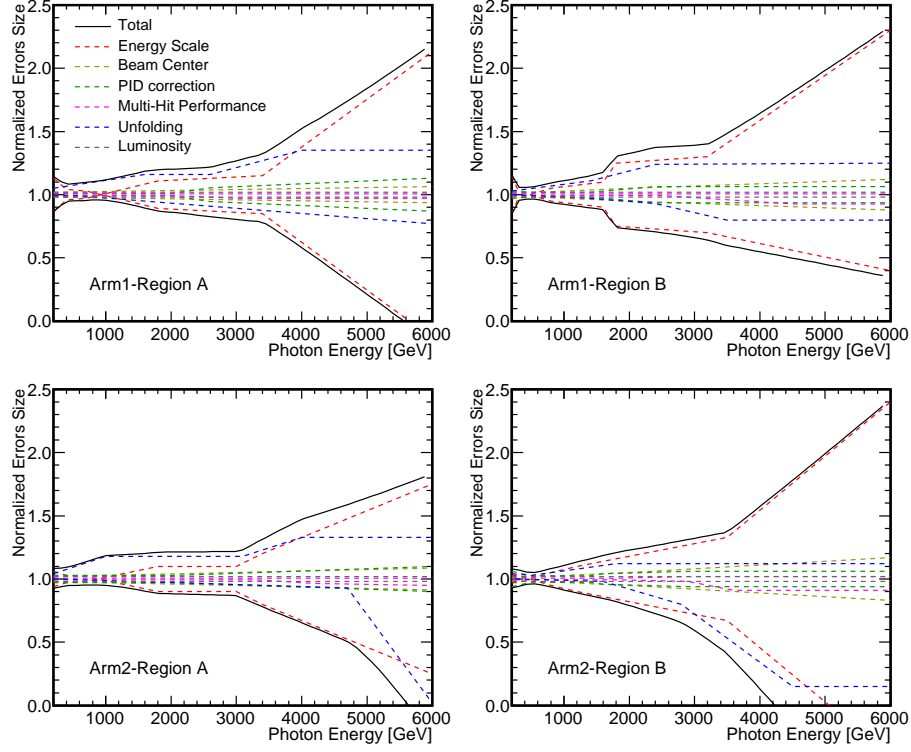


Figure 2: Systematic uncertainties of the photon spectra in the Arm1 (top) and Arm2 (bottom) analyses. The left and right panels correspond to the results of the two analysis regions. The colored and dashed lines indicate the estimated systematic uncertainties after normalization with the mean values of the experimental data. The black line indicates the total systematic uncertainties calculated as quadratic summations of the all uncertainties.

correlated between Arm1 and Arm2. These uncertainties were added quadratically to the combined results. The upper panels of Fig. 4 show the combined spectra with the predictions of the hadronic interaction models, QGSJET II-04, EPOS-LHC, DPMJET 3.06, SIBYLL 2.3, and PYTHIA 8.212. The hatched areas indicate the total statistical and systematic uncertainties, which were calculated using the combined method. The bottom panels show the ratio of MC predictions to the experimental results. In the pseudorapidity region  $\eta > 10.94$ , the QGSJET II-04 and EPOS-LHC models show the best agreement overall with the data. PYTHIA 8.212 shows good agreement with the data from the lowest

energy bin to near the 3 TeV bin although it clearly predicts higher flux than the data in the energy region greater than 3 TeV. DPMJET 3.06 and SIBYLL 2.3 predict flux higher and lower, respectively, than the data in most of the energy range. In the pseudorapidity region  $8.81 < \eta < 8.99$ , results from the EPOS-LHC and PYTHIA 8.212 models show good agreements with the data except at the high-energy end above 3 TeV. QGSJET II-04 and DPMJET 3.06 predict flux lower and the higher, respectively, than the data. SIBYLL 2.3 displays a different trend from the result in  $\eta > 10.94$ , predicting higher flux than the data in the energy range above  $> 1.5$  TeV. This result is related to the fact that SIBYLL 2.3 predicts a larger mean value of  $p_T$  for photons than both the data and the other models.

The general trends demonstrated by the data and MC simulations resemble the results obtained from proton-proton collisions at  $\sqrt{s} = 7$  TeV in Ref. [12], which showed the measured energy spectra for forward photons in the same pseudorapidity regions compared with MC predictions from QGSJET II-03, EPOS 1.99, SIBYLL 2.1, DPMJET 3.04, and PYTHIA 8.145. Except for DPMJET 3.04, these models are older versions than ones to which Fig. 4 refers. The updates to these models and the differences of collision energy do not produce significant changes in the forward-photon energy spectra in the QGSJET II and EPOS models. Thus, the detailed differences in the results from  $\sqrt{s} = 7$  TeV and  $\sqrt{s} = 13$  TeV may correspond to the differences between the  $p_T$  coverages.

## 7. Summary

The LHCf experiment measured the inclusive energy spectra of forward photons at  $\eta > 10.94$  and  $8.99 > \eta > 8.81$  with proton-proton collisions at  $\sqrt{s} = 13$  TeV. The two LHCf detectors, Arm1 and Arm2, gave consistent results and the results were combined while considering their statistical and systematic uncertainties. The final results were compared with the MC predictions obtained from several hadronic interaction models: QGSJET II-04, EPOS-LHC, DPMJET 3.06, SIBYLL 2.3, and PYTHIA 8.212. Among these models, EPOS-LHC

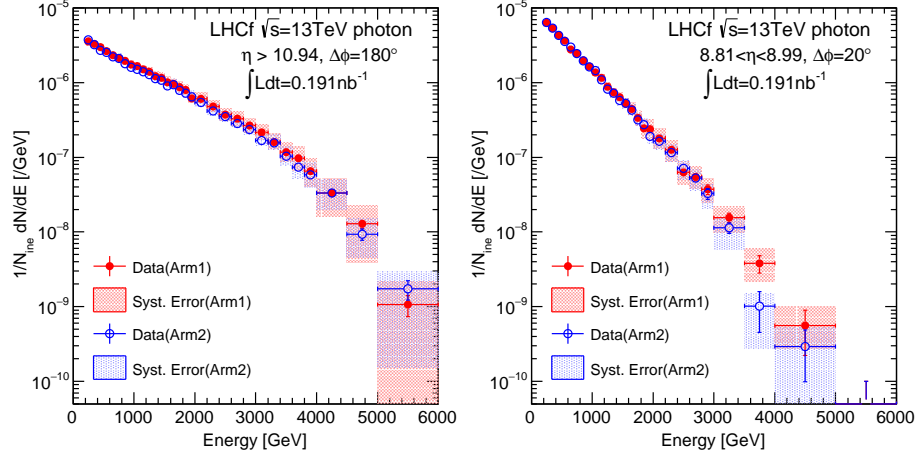


Figure 3: Photon spectra measured by the Arm1 (red filled circle) and Arm2 (blue open circle) detectors. The left figure shows the results for the region  $\eta > 10.94$ , which covers the zero degree of collisions. The right figure shows the results for the region  $8.81 < \eta < 8.99$ , which corresponds to the fiducial area in the large calorimeters of the detectors. The bars and the shaded areas correspond to the statistical and the systematic uncertainties, respectively. Only uncorrelated systematic uncertainties between Arm1 and Arm2 are considered in these plots.

showed the best agreement with the experimental data. QGSJET II-04 showed good agreement with the data for  $\eta > 10.94$  but predicted lower flux than the data for  $8.99 > \eta > 8.81$ . PYTHIA 8.212 showed the higher flux than the data in the energy region above 3 TeV.

No MC models matched the experimental data perfectly. The differences between the data and MC models were attributable to a less-than-complete understanding of the soft hadronic interactions implemented in the models as diffractive processes [29, 30]. Common operations of the LHCf with the ATLAS experiment, in which the detector covers the central region of IP1, were performed in 2015. The detailed studies with event-by-event information measured by ATLAS will be able to help us understand more fully the production of photons in the forward region [30].

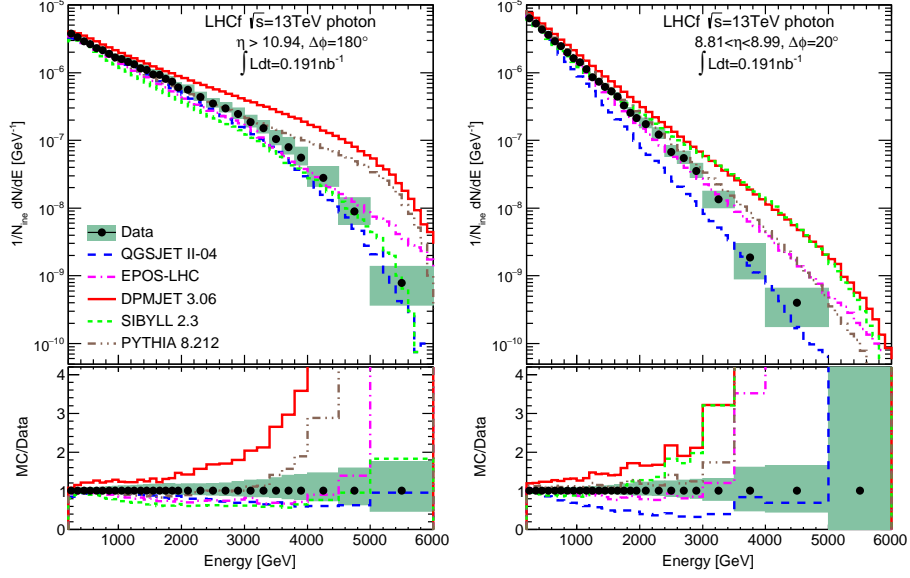


Figure 4: Comparison of the photon spectra obtained from the experimental data and MC predictions. The top panels show the energy spectra, and the bottom panels show the ratio of MC predictions to the data. The hatched areas indicate the total uncertainties of experimental data including the statistical and the systematic uncertainties.

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